Consequences of a Diagrammatic Representation of Paul Cohen's Forcing Technique Based on C.S. Peirce's Existential Graphs

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Abstract. This article examines the forcing technique developed by Paul Cohen in his proof of the independence of the Generalized Continuum Hypothesis from the ZFC axioms of set theory in light of the theory of abductive inference and the diagrammatic system of Existential Graphs elaborated by Peirce. The history of the development of Cohen's method is summarized, and the key steps of his technique for defining the extended model $M[G]$ from within the ground model M are outlined. The relations between statements in M and their correspondent reference values in $M[G]$ are modeled in Peirce's Existential Graphs as the construction of a modal covering over the sheet of assertion. This formalization clarifies the relationship between Peirce's EG-β and EG-γ and lays the foundation for theorizing the abductive emergence of the latter out of the former.

1 Introduction

C.S. Peirce's concept of abduction represents one of the very few fundamental and substantial innovations in the history of logic since Aristotle and the Stoics. In addition to the two broadest categories of logical inference – deduction and induction – Peirce proposes a third, abduction, which would share certain characteristics with both. The term abduction is meant to capture the specific form of inference involved in hypothesis-formation. As such, it intrinsically links logic, experimental method and intellectual creativity in a single process. To give some sense of the scope Peirce accorded to the concept,

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it may be sufficient to state that Peirce considered all forms of perceptual judgment to be species of abductive inference.

One question that remains essential to the debates over the status of abduction is whether abduction itself is susceptible to formalization and, more generally, as Hoffman inquires, “Is there a logic of abduction” at all [5]? Hoffmann’s reflections on Peirce emphasize the context-dependent character of all logic and representation, and he argues that the mechanism of abduction (if, indeed, mechanism is a fair attribution) may be best understood as a rearranging of contexts of knowledge, description and practice. Practice takes priority here – as Hoffman writes:

> A central condition for taking new perspectives is activity. Peirce emphasizes this element of activity in particular in respect to discoveries in mathematics: Proofs and deductive reasoning are essential characteristics of mathematics, but if we want to prove that the sum of angles in a triangle is exactly 180°, we need a form of reasoning which Peirce called “diagrammatic”... The essence of diagrammatic thinking is to create new representations out of a given one. The point is that *one* representation in a continuum of possible representations “compels us” to perceive new relations or a new organizing structure of a set of data.

In this way, the eminently practical character of abduction intersects with diagrammatic method, which will prove crucial in what follows.

We may identify two particular issues that arise in addressing the question whether there is a “logic of abduction”. The first is the specifically creative character of hypothesis-formation. Peirce himself emphasizes that one of the marks distinguishing abduction from both deduction and induction is that a new term appears in the conclusion of the abductive inference that cannot be found in either the major or minor premise. In this way, abduction introduces genuine novelty into thought, and it would seem that this precludes any formalization of its logic.

The second issue is the relation of abduction to Peirce’s synechistic conception of logic and ontology. In his later work, Peirce stresses that the nature of thought cannot be adequately grasped without reference to continua [7]. In particular, he took tentative steps towards a topological redefinition of logic. It is not clear if and how this development in Peirce’s mature thought recontextualizes the problem of abduction.

Both of these issues lead us to pose the question of a possible logic of abduction in more specific terms, namely with respect to the system of Existential Graphs which Peirce himself identified as his most important contribution to logic.

The purpose of this work is not definitively to answer the question of the formalization of abduction. Rather, we provide an iconic representation of the forcing relation – or, more precisely, the relationship between this relation itself and its reference values in the extended model – in Peirce’s EG. In particular, the *forcing* relation as defined by Cohen between *conditions* and statements S, denoted by

$$\pi \models S$$

which may be expressed in Peirce's EG at the $\beta-$level, serves as the basis for the indication of modal statements in EG-γ which apply to "possibilia" in an extended universe of reference. This representation of forcing in EG, in conjunction with Badiou's extended argument that Cohen's mathematics is the formal context within which hypothesis formation and generic truth must be framed, lays the foundation for future work in the formalization of abductive inference within EG[1].

2 The Continuum Hypothesis

Peirce's contemporary G. Cantor formulated what has come to be called the Continuum Hypothesis. The technique that we will be using as the *trait d'union* between abduction and the diagrammatic representation of the emergence of EG-γ from EG-β was developed by Paul Cohen in his work to prove the independence of this hypothesis from the Zermelo-Frænkel axioms of set theory. What follows is a brief summary.

Let us denote by ω the set $\omega = \{0, 1, 2, \ldots\}$, by $|S|$ the cardinality of a set S, by $\aleph_0$ the cardinality of ω and by $\mathcal{P}(S)$ the power set of S. Then the Continuum Hypothesis (abbreviated CH) is the conjecture, formulated by Cantor, that, if S is an infinite subset of $\mathcal{P}(\omega)$, then either $|S| = \aleph_0$ or $|S| = \mathcal{P}(\omega)$. In particular, CH asserts that there is no set whose cardinality is strictly between that of the integers and that of the real numbers.

This conjecture was the first on the list of problems presented by David Hilbert in the year 1900. About thirty years later, Kurt Gödel [4] proved that CH cannot be disproved from the Zermelo-Frænkel set of axioms (abbreviated ZF) and that therefore, if ZF is consistent, ZF+CH is still consistent.

More precisely, Gödel defined the class of *constructible* sets and showed that the statement (which, interestingly, he denoted simply with the letter "A"):

Every set is constructible

is consistent with ZF and, moreover, that CH can be proved from ZF as restricted to constructible sets and therefore, that ZF+CH is consistent.

The intrinsic difficulties in constructing alternate models of the Zermelo-Frænkel axioms was a serious obstruction towards any substantial progress about CH. It was not until the early 1960s that Paul Cohen found a method to construct models of ZF in which neither proposition A, nor CH holds. Combined with Gödel's results, Cohen showed that CH is *independent* from ZF.

[1] The authors would like to thank Ahti-Veikko Pietarinen, Jaakko Hintikka and Priscila Borges for their comments and suggestions at the MBR 2009 Symposium on Peirce's Existential Graphs.

Cohen's work relies on a novel technique that he invented *ex novo*, called *forcing*. In what follows we will outline the fundamental, simple ideas behind forcing and discuss how this method constitutes a natural framework for the emergence of both of Peirce's "Logic of Continuity" as expressed in EG$-\gamma$ and abduction.

3 The Generic Extension of Standard Models of ZF

The following section is a brief overview of the background preparation necessary to define forcing. Our presentation follows the structure that Badiou [1] uses in his own summary of Cohen.

Cohen's technique of forcing makes use of two models of ZF, one of which is taken to be the "ground model", which we will denote by M, and its *extension* $M[G]$ obtained by adjoining what will be called a *generic set* G to M. G is *generic* in the sense that it remains indiscernible from the point of view of what is codable within M, so that M and $M[G]$, while both models of ZF, function at distinct levels. Forcing will make use of this difference. The precise character of this difference is elaborated in the following sequence of steps.

3.1 The Generic Set

A model for the axioms of set theory can be simply thought as a "realization" of those axioms, exactly in the same way as a group, say $(\mathbb{Z}, +)$ (the group of the integers with the operation of addition), is a "realization" for the group axioms.

The goal of this subsection is to define a notion of *generic set*: intuitively, a set G is generic with respect to a model M, if "from inside" M, G can be defined but yet cannot be discerned. In order to unfold this only apparent contradictory definition, we need to outline precisely the steps of the construction. In what follows we only assume familiarity with the basic notation of set theory.

3.1.1 Standard Models

We start with a *standard* model M for ZF, where by *standard* we mean that:

- every element x of M is a well-founded set: that is, x is constructed inductively from the empty set using operations such as taking unions, subsets, powersets, etc.;
- M is transitive: every member of an element of M is also an element of M;
- M is countable.

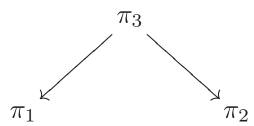
3.1.2 Set of Conditions

Within M, we discern a set of *conditions*, that we denote by $\mathcal{C}$, such that:

- $\mathcal{C} \in M$;
- there is a partial order on $\mathcal{C}$, denoted by $\subset$.

We say that π_1 *dominates* π_2 if $\pi_2 \subset \pi_1$ and that π_1 is compatible with π_2 if there is π_3 such that $\pi_1 \subset \pi_3$ and $\pi_2 \subset \pi_3$. If we represent the order with the branches of a tree as below

then this can be expressed graphically by the following diagram:

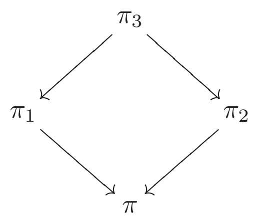


Then the last property that identifies $\mathcal{C}$ is that

- every condition is dominated by two conditions which are incompatible among themselves:

$$\forall \pi\ \exists \pi_1, \pi_2 \text{ such that } (\pi \subset \pi_1)\ \&\ (\pi \subset \pi_2) \text{ with } \pi_1 \text{ and } \pi_2 \text{ incompatible.}$$

Graphically, that means that, if π_1 and π_2 are incompatible, then a picture like the following is impossible within $\mathcal{C}$:



Example 1. Let $\mathcal{C}$ be the set of all the finite binary sequences. That is

$$\mathcal{C} = \bigcup_{n \in \mathbb{Z}^+} < a_1, a_2, \ldots, a_n >$$

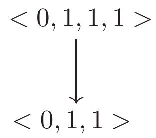
where $a_i \in \{0, 1\}\ \forall i \in \{1, 2, \ldots, n\}$. Let the order be defined by

$$< a_1, a_2, \ldots, a_n > \subset < b_1, b_2, \ldots, b_k >$$

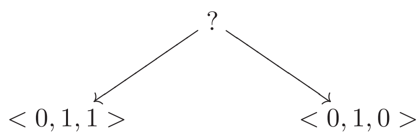
if and only if the first n entries of $< b_1, b_2, \ldots, b_k >$ coincide with $< a_1, a_2, \ldots, a_n >$; that is if and only if

1. $n < k$
2. $a_i = b_i \ \forall i \in \{1, 2, \ldots, n\}$

In this case we have, for instance, that $< 0, 1, 1 > \subset < 0, 1, 1, 1 >$. Graphically:



By the same token, it easy to see that, for instance, the sequences $< 0, 1, 1 >$ and $< 0, 1, 0 >$ are incompatible because there does not exist a sequence which dominates both:



3.1.3 Correct Subsets of $\mathcal{C}$

Within $\mathcal{C}$, we further discern a set δ, called a *correct* set of conditions, such that:

1. $(\pi_2 \in \delta, \pi_1 \subset \pi_2) \Rightarrow \pi_1 \in \delta$;
2. $(\pi_1 \in \delta, \pi_2 \in \delta) \Rightarrow \exists\ \pi_3 \in \delta$ such that $\pi_1 \subset \pi_3, \pi_2 \subset \pi_3$.

This definition is reminiscent of that of an ideal in ring theory – in our case an element of a correct set δ is such that it forces every object that is below it in the order to belong to δ.

Example 2. Following up on the previous example, let $\mathcal{C}$ be again the set of all the finite binary sequences with the order defined above. It can be checked that the subset of $\mathcal{C}$ whose elements are the finite sequences having only $1'$s as entries is a correct set (let us call such a subset δ_1). Indeed, only finite sequences of all $1'$s can be dominated by sequences of the same kind and two such sequences are clearly compatible (it is enough to take a third sequence of all 1′s longer than the two chosen ones).

Once we realize that a correct set can be discerned in an unambiguous way by its defining property (i.e. sequence of all 1's), it is remarkable to notice that the interplay between the structure of $\mathcal{C}$ and that of δ generates a natural boundary between the discernible and its complement. Indeed, since any element, say π_1 in $\mathcal{C}$ must be dominated by two incompatible conditions, say π_2 and π_3, by the property 2 above for δ at least one of them, say π_2, cannot be an element of δ. Therefore π_2 does not possess the property which discerns δ!

Badiou cleverly notes that "the concept of correct set is perfectly clear for an inhabitant of M...". What he means by this is that the correct set can be discerned from within M: a suitable formal language that is codable in M is powerful enough to "see" the crisp boundaries of δ within $\mathcal{C}$. Badiou continues, "What is not yet known is how to describe a correct set which would be an indiscernible part of $\mathcal{C}$, and so of the model M" [1]. This is our next task.

3.1.4 Dominations

We start with an example which reconnects with those given above ($\mathcal{C}$ is again the set of all the finite binary sequences and the order is the usual one).

Example 3. The property discerning the complement of δ_1 in $\mathcal{C}$, say δ_1^c is given by: "binary sequences containing at least one 0". For any element in δ_1 there will be an element in δ_1^c which dominates it. But this offers a beautiful way to let the discernibility of δ_1 be characterized in a structural way, with no reference to the language: for any element of δ_1, for instance $< 1, 1, 1 >$ there is at least one element in the complement, say $< 1, 1, 1, 0 >$ which dominates it. This only apparently innocent observation is at very heart of the foundation of the generic.

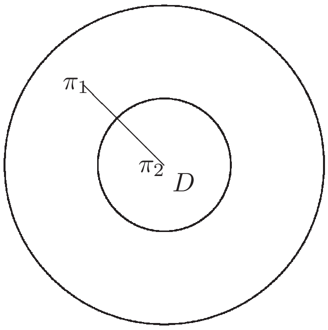
To formalize what we were just discussing in this example, consider a correct set δ. δ is *discernible* from an inhabitant of M if there is an explicit property λ (expressible within the model M) that distinguishes δ unambiguously:

$$\alpha \in \delta \iff \lambda(\alpha)$$

Since every condition $\pi_1 \in \delta$ is dominated by two incompatible conditions π_2 and π_3, by the second property defining δ we have that either π_2 or π_3 has to live outside δ. The importance of this remark cannot be overestimated. It is indeed upon this simple observation that we can define what a *domination* is. Let's define a *domination* D as a set of conditions such that any condition outside the domination is dominated by at least one condition inside the domination. In symbols:

$$\sim (\pi_1 \in D) \Rightarrow (\exists \pi_2)[(\pi_2 \in D)\&(\pi_1 \subset \pi_2)]$$

(here $\sim$ denotes negation).



3.1.5 The Generic Set

We may now define a generic set. A correct set G will be said generic for M if, for any domination D which belongs to S, we have $D \cap G \neq \emptyset$. In other words, G is a correct set that has the property of having at least one element in common with all the dominations. From the point of view of M, it should be clear why G is indiscernible: "…otherwise it would not intersect the domination which corresponds to the negation of the discerning property" [1].

3.2 From M to M[G]

Let us remember that our goal is to find another model of ZF by *adjoining* G to M. In order to accomplish this task, we need two more intermediate steps.

3.2.1 Names

In spite of its indiscernibility, elements of G must be given names from within M. This is done by using a technique very close, in spirit, to transfinite induction. We begin by defining names μ of *rank 0*:

$$rank(\mu) = 0 \iff [(\gamma \in \mu \Rightarrow \gamma =< \emptyset, \pi >)]$$

where $\pi \in \mathcal{C}$ is any condition. Essentially, names of rank 0 are thus (any possible) sets of ordered pairs, of which each ordered pair simply indexes a particular condition (an element of $\mathcal{C}$) to the empty set.

Inductively we can then define names of rank greater than 0:

$$rank(\mu) = \alpha \iff [(\gamma \in \mu \Rightarrow \gamma =< \mu_1, \pi >)] \ \& \ rank(\mu_1) < \alpha.$$

In effect, this defines an inductive hierarchy of names in which some name at a given ordinal rank greater than 0 is itself constituted as a set of ordered pairs, of which each ordered pair indexes a particular condition to some name as determined at a lower rank. These names, whose ordinal induction may be entirely defined from within M but which cannot in fact all be themselves elements of M, will be used to produce the "excess" of $M[G]$ over and above M by assigning a specific value to each name according to a second inductive procedure.

3.2.2 Reference Values

Once we can name elements of G, we want to assign *referential values* to these names which will in effect determine the essential features of the adjunction of G to M through the action of G (or rather, the strictly formal definition of G from within M as outlined above in section 3.1) upon the ordinal hierarchy of names. In this way, the names will be used to indicate the generic set G sufficiently without thereby discerning or fully determining it. We start by defining referential values for names of rank 0: For names of rank 0 (which are composed of pairs $<\emptyset,\pi>$) we posit:

- $R_G(\mu)=\{\emptyset\} \iff \exists <\emptyset,\pi>\in\mu \mid \pi\in G$
- $R_G(\mu)=\emptyset$ otherwise.

Inductively, let us suppose that the referential value of the names has been defined for any rank less than α. Then, if μ_1 is a name such that $rank(\mu_1)=\alpha$, we have that:

$$R_G(\mu_1)=\{R_G(\mu_2) \mid (\exists\pi)(<\mu_2,\pi>\in\mu_1 \ \&\ \pi\in G)\}$$

Each of the names is thus formally assigned a unique value that depends solely upon which conditions are indeed elements of G in any given case, although G itself remains relatively undetermined (save for its genericity).

3.2.3 Adjoining the Generic Set to the Ground Model

At this point we are in a position to define $M[G]$:

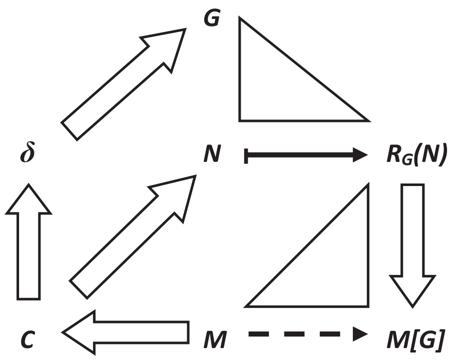
$$M[G]=\{R_G(\mu) \mid \mu\in M\}$$

In plain words, $M[G]$ is the set of all the G-referential values of names that are themselves elements of M. We will not go into further technical details, but it is worthwhile, for the sake of clarity, to mention that the following, fundamental facts can be proven:

- $M\subset M[G]$, that is, $M[G]$ is a non-trivial extension of M
- $G\in M[G]$ and, moreover, G itself can be named from the point of view of M.

3.3 Diagrammatic Recapitulation

The diagram below summarizes the order and connections between the various steps of the construction of $M[G]$. The two triangles represent determinations involving more than one of the previous results.



4 Modeling the Forcing Relation in EG

Cohen's mathematical technique of forcing produces a correspondence between a well defined forcing relation in M and possible truth values of statements in $M[G]$. We represent this correspondence iconically in Peirce EG as a lifting of EG-β statements on a single sheet of assertion into a modal sheet of EG-γ. In our diagrammatic construction, discrete modal statements at the level of the EG-β sheet of assertion take as their referents the emergent continuous domain of $M[G]$ as expressed in EG-γ.

4.1 The Quasi-implicational Structure of Forcing and Peirce's Existential Graphs

On the one hand Cohen, in his exposition of forcing, highlights its modal nature, noting that

> Clearly there are some properties of S which no reasonable procedure could interpret as being true or false for a 'generic' set G... Given π we will then ask, if under some procedure to be given, it is reasonable to expect that π forces a statement S about G to hold or forces $\sim S$ to hold, or whether the condition π does not force S one way or the other. *Although forcing will be related to the notion of implication it will differ from it in that given that π forces S it will not be true that any G that satisfies π will also satisfy S. What will be true is that any generic G satisfying π will also satisfy S* [notation altered to conform with the presentation above and emphasis added].

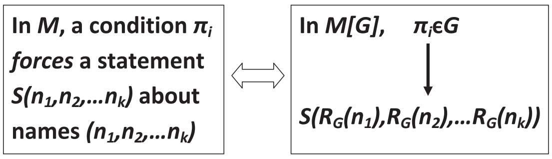
On the other hand, in his system of Existential Graphs, Peirce distinguishes between the single sheet of assertion at the level of α and β graphs and a "book of separate sheets, tacked together at points" at the $\gamma-$level, where modality emerges from relations of continuity and discontinuity between the sheets.

> In the gamma part of the subject all the old kinds of signs take new forms... Thus in place of a sheet of assertion, we have a book of separate sheets, tacked together at points, if not otherwise connected. For our alpha sheet, as a whole, represents simply a universe of existent individuals, and the different parts of the sheet represent facts or true assertion concerning that universe. At the cuts we pass into other areas, areas of conceived propositions which are not realized. In these areas there may be cuts where we pass into worlds which, in the imaginary worlds of the outer cuts, are themselves represented to be imaginary and false, but which may, for all that, be true, and therefore continuous with the sheet of assertion itself, although this is uncertain (cited in [9]).

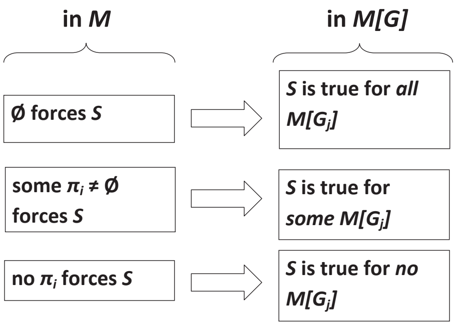
Forcing is intrinsically relational – it distributes a determination or "control" across two heterogeneous domains. $M[G]$ is built up solely from resources available in M, but the very genericity of G guarantees that the extended model $M[G]$ is indiscernable from the standpoint of M and thus radically different or "Other". This is why the definition of G on the basis of the dominations is so important – dominations effectively formalize an intuitively epistemological or semiotic concept, namely the discernability of a "property" of some given correct set. It is the power of the generic to name the indiscernible [1] that corresponds to the creative moment that Peirce emphasizes is central in the logical operation of abduction.

The second crucial element of the heterogeneity of M and $M[G]$ is the pair of transfinite inductions that determine the hierarchy of names on the one hand and the reference-values on the other. Notice that the names are defined independently of G while the reference-values are determined precisely by coordinating names with elements of G. If the names function first of all as indices of conditions and then, through the succession of ordinal ranks, indices of possible collections of these indices as paired with conditions, and so on, the reference-values function as the well-defined link that connects the names in M to elements of $M[G]$.

Forcing is, quite simply, a carefully-defined relation between conditions and statements in M. What matters of course is precisely how this relation is defined: the effort of building up the extended model $M[G]$ in the way outlined above becomes worthwhile solely because it enables this definition itself. Forcing is a relation in M between a given condition and a statement about the names that holds if and only if the corresponding statement about the reference-values of those names is verifiable in $M[G]$ and the given condition is an element of G.



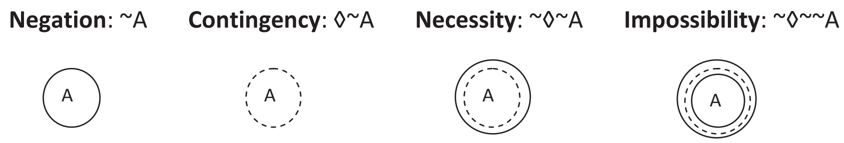
Thus, a relation that is wholly determined within M is able to signify the verifiability of statements about the extended model $M[G]$, depending upon whether or not particular conditions in M are also elements of G. Because of this, it becomes entirely determinable within M given a statement S whether or not the condition of the empty set forces S, whether or not some but not all conditions force S, and whether or not no conditions whatsoever force S. Indeed it is easily shown that these three cases or possibilities are both mutually exclusive and exhaustive. To each of the three cases corresponds a specific kind of knowledge about the statement S which takes the reference-values of the names as its argument, and hence is a statement about $M[G]$. The following table lays out the relevant correspondences:



As this table shows, forcing is thus both an explicit relation in M (between conditions and statements) and an implicit correlation between M and $M[G]$ (determining at least some knowledge of the latter from the standpoint of the former, namely whether the corresponding statement about $M[G]$ is always, sometimes but not always or never verifiable).

4.2 Diagramming Forcing as the Abductive Emergence of EG−γ from EG−β

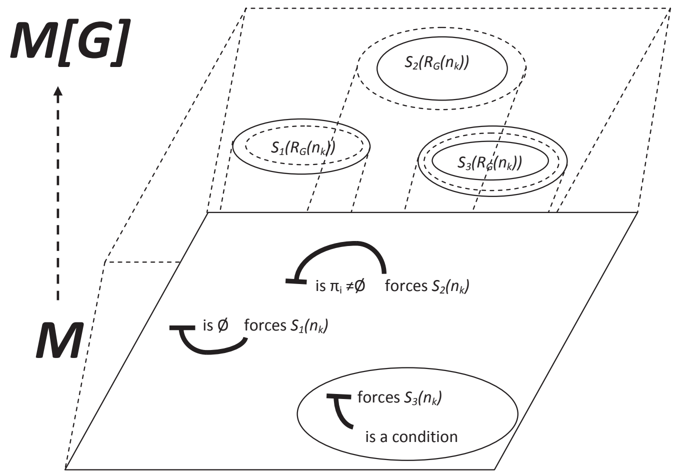
Within the newly constituted horizon of statements about $M[G]$ from the standpoint of M, the three “cases” may therefore be interpreted in terms of the three logical modalities expressible in Peirce’s EG-gamma: necessity, contingency and impossibility. Peirce’s EG-γ notation is as follows:



An essential aspect of abduction is that its conclusions, like those of induction, are at best merely probable. Yet even more strongly, Peirce insists that abduction is not even determinately probable – this is due directly to its “creative” element. It is exactly this character of abduction that has eluded formalization.

Yet the modalities expressible in EG-γ reflect the bare, minimal knowledge that is both necessary and sufficient for abduction. Thus if we are able to express formally the relation between non-modal EG-β statements in M and modal EG-γ statements in $M[G]$, then we will have within Peirce’s graphical system a diagrammatic representation of the specific correlational structure of forcing insofar as it defines modal truth-values in $M[G]$ from within M.

The following diagram represents, in EG, the lifting of the forcing relations in the ground model M into the modal truths of $M[G]$ (for the sake of concise representation, n_k here represents the sequence $n_1, n_2, \ldots, n_k$):



This representation shows how the set-theoretical statements and relations of forcing may be translated into the formal-iconic language of EG. This form itself is productive of new content: the EG-γ representation of forcing is itself a diagram whose internal relations carry additional implications and consequences. In particular, it illustrates the abductive character of forcing from an essentially topological perspective. In this way, we produce a rigorous framework for the further investigation of the problem of a logic of abduction in terms of the problematic of the topological relations between the continuum and its possible coverings.

5 Conclusions

Our formalization of the Peircean abductive structure of forcing extends the philosophical interpretation of Cohen's work offered by Badiou in [1] while remaining fully within the formal language of EG. Badiou understands the universe of Gödel's constructible sets to correspond to Being qua Being whereas the models of ZFC that adjoin generic sets in accordance with Cohen's technique indicate a domain in which infinite procedures of truth may exceed Being as such. As Badiou himself points out, such "truth procedures" rely upon the formation of hypotheses that are always necessarily in excess of what has already been given, but are expressed in terms of the "finite fragment made up of the present state of the enquiries":

> ... the subject solely controls – because it is such – the finite fragment made up of the present state of the enquiries. All the rest is a matter of confidence, or of knowing belief. Is this sufficient for the legitimate formulation of a hypothesis of connection between what a *truth* presents and the *veracity* of a statement that bears upon the names of a subject-language? Doesn't the infinite incompletion of a truth prevent any possible evaluation, *inside* the situation, of the veracity to-come of a statement whose referential universe is suspended from the chance, itself to-come, of encounters, and thus of enquiries?

Cohen's proof demonstrates mathematically that the answer to this question is no. The forcing relationship in M allows us to make definite, but limited, assertions about truths in $M[G]$. For Badiou, this relationship between the discretely statable and the generically indiscernible instantiates what he calls the *fundamental law of the subject*:

> if a statement of the subject-language is such that it will have been veridical for a situation in which a truth has occurred, this is because *a* term of the situation exists which both belongs to that truth (belongs to the generic part which *is* that truth) and maintains a particular relation with the names at stake in the statement.

Our diagram formalizes the relationship in question as the emergence in Peirce's EG-β of a continuous sheet in excess of the sheet of assertion which corresponds to the reference-domain of $M[G]$.

In his seminal paper, Louis Kauffman [6] establishes deep connections between Peirce's EG and infinitesimals from the topological perspective that naturally emerges from Peirce's ideas. We hope that our work can contribute to opening new ground for the construction and investigation of a formal topological theory of continuous modalities and abductive processes.

References

1. Badiou, A.: Being and Event. O. Feltham(trans.) Contiuum, London, New York (2005)
2. Chow, T.: A beginner's guide to forcing. Contemporary Mathematics 479, 35–40 (2009)
3. Cohen, P.: Set Theory and the Continuum Hypothesis. Dover Publications, Mineola (1966)
4. Gödel, K.: The Consistency of the Continuum Hypothesis. Princeton University Press, Princeton (1940)
5. Hoffmann, M.: Is There a 'Logic' of Abduction?, `http://user.uni-frankfurt.de/~wirth/texte/hoffmannabdu.htm`
6. Kauffman, L.: The mathematics of Charles Sanders Peirce. Cybernetics & Human Knowing 8(1-2), 79–110 (2001)
7. Peirce, C.S.: Reasoning and the Logic of Things. In: Ketner, K.L. (ed.), Harvard University Press, Cambridge (1992)
8. Roberts, D.D.: The Existential Graphs of Charles S. Peirce. Mouton and Co., The Hague (1973)
9. Zalamea, F.: Peirce's Logic of continuity: Existential graphs and non-cantorian continuum. Review of Modern Logic 9(29), 115–162 (2003)